Short talks

EVEQ 2024, NextGen

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Shape sensitivity of a blood damage model

(joint work with John Sebastian H. Simon)

Valentin Calisti
Institute of Mathematics, Czech Academy of Sciences

Monday 16:15-16:30

In this work, we optimize the shape of a 2D domain containing a blood flow, with the aim of minimizing blood damage. The blood is modeled by unsteady incompressible Navier-Stokes equations, with appropriate inlet and outlet boundary conditions. In general, in the literature, blood damage indices depend on the blood velocity and a scalar measure of the viscous stress. We model their evolution by a reaction-diffusion-advection equation. We analyze the shape sensitivity of this problem with a rearrangement method. This enables us to numerically optimize the shape of a rigid obstacle in the blood flow.

On the steady compressible Navier–Stokes–Fourier system with temperature dependent viscosities

Milan Pokorný
Charles University

Monday 16:30-16:45

The presentation is based on a joint work with O. Kreml (Academy of Sciences, Czech Republic, Prague), T. Piasecki (University of Warsaw, Poland) and Emil Skříšovský (Charles University, Prague). We consider the system of partial differential equations in $\Omega \subset \mathbb{R}^3$

$$\operatorname{div}(\varrho\mathbf{u}) = 0, \tag{1}$$

$$\operatorname{div}(\varrho\mathbf{u}\otimes\mathbf{u}) - \operatorname{div}\mathbb{S} + \nabla p = \varrho\mathbf{f}, \tag{2}$$

$$\operatorname{div}(\varrho E\mathbf{u}) = \varrho\mathbf{f}\cdot\mathbf{u} - \operatorname{div}(p\mathbf{u}) + \operatorname{div}(\mathbb{S}\mathbf{u}) - \operatorname{div}\mathbf{q} \tag{3}$$

which describes steady flow of a heat conducting compressible fluid in a bounded domain Ω. We consider (1)–(3) together with the no-slip or slip boundary conditions for the velocity at $\partial\Omega$

$$\mathbf{u}\cdot\mathbf{n} = 0, \tag{4}$$

$$\mathbf{u}\cdot\boldsymbol{\tau} = \mathbf{0} \qquad \text{or} \qquad (\mathbb{S}\mathbf{n})\cdot\boldsymbol{\tau} + \lambda\mathbf{u}\cdot\boldsymbol{\tau} = \mathbf{0}, \tag{5}$$

and Newton's type boundary conditions for the temperature

$$-\mathbf{q}\cdot\mathbf{n} + L(\vartheta - \Theta_0) = 0 \tag{6}$$

or Dirichlet boundary condition

$$\vartheta = \vartheta_D. \tag{7}$$

We assume the fluid to be Newtonian, i.e. $\mathbb{S} = \mathbb{S}(\vartheta,\nabla\mathbf{u}) = \mu(\vartheta)(\nabla\mathbf{u} + \nabla\mathbf{u}^T - \frac{2}{3}\operatorname{div}\mathbf{u}\mathbb{I}) + \xi(\vartheta)\operatorname{div}\mathbf{u}\mathbb{I}$, with the pressure $p(\varrho,\vartheta) \sim \varrho\vartheta + \varrho^\gamma$ and the heat flux $\mathbf{q}(\vartheta,\nabla\vartheta) = -\kappa(\vartheta)\nabla\vartheta$. We study existence of a solution to our problem (1)–(6) or (1)–(5), (7) in dependence on γ, α and m, where $\mu(\vartheta)$, $\xi(\vartheta) \sim (1+\vartheta)^\alpha$, $\kappa(\vartheta) \sim (1+\vartheta)^m$.

Motion of a heat conducting compressible fluid in moving domain with nonhomogeneous boundary

Kuntal Bhandari
Institute of Mathematics, Czech Academy of Sciences

Monday 16:45-17

We consider a flow of heat conducting compressible fluid inside a moving domain whose shape in time is prescribed. The flow is governed by the 3-D Navier-Stokes-Fourier system where the velocity is supposed to fulfil the full-slip boundary condition and the temperature on the boundary is given by a nonhomogeneous Dirichlet condition. We establish the global-in-time weak solution to the system. To accommodate the nonhomogeneous boundary heat flux, we introduce the concept of ballistic energy in the weak formulation.

Oberbeck-Boussinesq and the boundary issue

Florian Oschmann
Institute of Mathematics, Czech Academy of Sciences

Tuesday 16:15-16:30

The Rayleigh-Bénard convection problem deals with the motion of a compressible fluid in a tunnel heated from below and cooled from above. In this context, the so-called Boussinesq relation is used, claiming that the density deviation from a constant reference value is a linear function of the temperature. These density and temperature deviations then satisfy the so-called Oberbeck-Boussinesq equations. The rigorous derivation of this system from the full compressible Navier-Stokes-Fourier system was done by Feireisl and Novotný for conservative boundary conditions on the fluid's velocity and temperature. In this talk, we investigate the derivation for Dirichlet boundary conditions, and show that differently to the case of conservative boundary conditions, the limiting system contains an unexpected non-local temperature term. This is joint work with Peter Bella (TU Dortmund) and Eduard Feireisl (CAS).

On the attractor and stability of NS with the dynamic slip boundary condition between two parallel plates

Michael Zelina
Charles University
Tuesday 16:30-16:45

We consider a fluid in the space between two infinitely long parallel plates. It is described by usual incompressible Navier-Stokes equations together with the dynamic slip boundary condition on the upper plate, i.e.

$$\beta \partial_t \boldsymbol{u} + \alpha \nu \boldsymbol{u} + \nu [(2D\boldsymbol{u})\vec{n}]_\tau = \beta \boldsymbol{h},$$

where α, β, $\nu > 0$. We will talk about an upper bound of the fractal dimension of the global attractor together with a needed higher regularity. Also, we shortly discuss the linearization principle and the stability of dynamic slip analogue of Couette flow. It is an extension of previous work with Dalibor Pražák in the setting of bounded domains.

Stability of Optical Solitons in 2d

Sergio Moroni
Basque Center of Applied Mathematics
Thursday 16:15-16:30

Optical properties of nematic liquid crystals have received great attention in the last years, as they can support stationary optical waves. Due to its high susceptibility, the response of a nematic liquid crystal to a light beam propagating through it is nonlocal and nonlinear. This response has a self-focusing effect on the light beam, supporting waveguides that counterbalance the diffraction spreading nature of light beam, and, in optimal shapes, allows the existence of stationary waves.

We study the ground states of the Schrödinger-Poisson system in dimension $(2+1)$

$$i\partial_z u + \frac{1}{2}\Delta u + u\sin(2\theta) = 0 \tag{8}$$

$$-\nu\Delta\theta + q\sin(2\theta) = 2|u|^2\cos(2\theta) \tag{9}$$

that models the propagation of a laser beam through a planar cell filled with a nematic liquid crystal.
The axis z, referred to as the optical axis, is the direction of the propagation of a light beam, while Δ is the Laplacian in the transverse coordinates (x, y).

Equation (8) represents the evolution of the light beam, with $u : \mathbb{R}^2 \to \mathbb{C}$ the complex amplitude of the electric field, while (9) is the nonlocal response of the medium, with $\theta : \mathbb{R}^2 \to \mathbb{R}$ the director field angle of the light-induced reorientation of crystal liquid molecules; q, ν are positive constants depending on the physics of the experiment.

In [2] a heuristic derivation of the equations is presented in the Appendix, while [1] and references therein give a more precise physical derivation.

The system was rigorously studied in [2], where the authors proved global existence and regularity for the Cauchy problem, and existence of stationary waves as minimizers, over the couples $(u, \theta) \in H^1 \times H^1$ with L^2 norm of u fixed, of the Hamiltonian:

$$E(u, \theta) := \frac{1}{4} \int_{\mathbb{R}^2} |\nabla u|^2 + \nu |\nabla \theta|^2 - 2|u|^2 \sin(2\theta) + q(1 - \cos(2\theta))\, dx \tag{10}$$

We present a first stability result for those stationary waves. This provides a strong justification of the relevance of the mathematical model to applications, as only locally stable solutions are expected to be seen in experiments and numerical simulations. Our main result is

Theorem 1 *Let (v, ϕ) be the configuration of minimal energy E over the constraint*

$$S_a := \left\{ (u, \theta) \in H^1 \times H^1 \quad | \quad \|u\|_{L^2}^2 = a \right\}$$

Then (v, ϕ) is orbitally stable with respect to the evolution (8)-(9).

As a definition of orbital stability we ask, loosely speaking, that the evolution through equations (8)-(9) of an initial datum close to v in H^1 remains close, modulo the symmetries of the Hamiltonian, to the ground state for all times.

In our proof, we adapt to the coupled system the arguments of [3], where stability is obtained from the positivity of the second derivative of the action. We prove at first a new estimate for perturbed configurations close to the ground state, showing that perturbation of the angle θ are controlled in norm by the associated perturbation of the light beam, up to a constant that depends on the shape of the ground state.

This allows to focus the stability study mainly on to the variable u. Hence we show positivity explicitly by Taylor expansion and minimization properties of the ground state.

References

[1] G. Assanto, N.F. Smyth: *Self-confined light waves in nematic liquid crystals* Physica D (2019) 132182

[2] J. P. Borgna, Panayotis Panayotaros, D. Rial, C. S. F. de la Vega 3: *Optical solitons in nematic liquid crystals: model with saturation effects* Nonlinearity 31, 1535-1559 (2018)

[3] M. Grillakis, J. Shatah, W. Strauss : *Stability theory of solitary waves in the presence of symmetry, I* Journal of Functional Analysis 74, (1987)

On a compressible fluid-structure interaction problem with slip boundary conditions

Šárka Nečasová
Institute of Mathematics, Czech Academy of Sciences
Thursday 16:30-16:45

This is a joint work with Yadong Liu (Nanjing Normal University, Nanjing, China) and Sourav Mitra (Indian Institute of Technology Indore,Indore, India). We study a system describing the compressible barotropic fluids interacting with (visco) elastic solid shell/plate. In particular, the elastic structure is part of the moving boundary of the fluid, and the Navier-slip type boundary condition is taken into account. Depending on the reference geometry (flat or not), we show the existence of weak solutions to the coupled system provided the adiabatic exponent satisfies $\gamma > \frac{12}{7}$ without damping and $\gamma > \frac{3}{2}$ with structure damping, utilizing the domain extension and regularization approximation. Moreover, via a modified relative entropy method in time-dependent domains, we prove the weak-strong uniqueness property of weak solutions. Finally, we give a rigorous justification of the incompressible inviscid limit of the compressible fluid-structure interaction problem with a flat reference geometry, in the regime of low Mach number, high Reynolds number, and well-prepared initial data.

References

[1] . Liu, S. Mitra, Š. Nečasová, *On a compressible fluid-structure interaction problem with slip boundary conditions*, Preprint 2024.

Capillary fluids, some applications

Matteo Caggio
Institute of Mathematics, Czech Academy of Sciences
Thursday 16:45-17

We present some recent applications in the context of capillary fluids. Particular attention will be given to weak-strong uniqueness results and highly and low compressible regimes.
